Principles of Math 12
Name:________________

Trigonometry Review Assignment:

1. Convert 100˚ to radians.

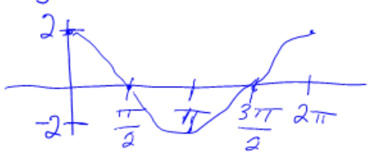
A. 0.18 B. 0.57 C. 1.75 D. 5.66

$100^\circ \times \frac{\pi}{180^\circ} = \frac{100\pi}{180} = \frac{10\pi}{18} = \frac{5\pi}{9} = 1.75$

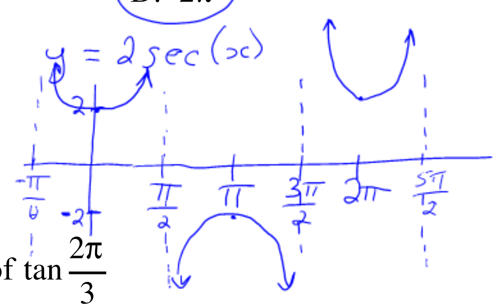
2. Give the period of y=2 sec(x)

A. $\frac{1}{2\pi}$ B. $\frac{\pi}{2}$ C. π D. 2π

$y = 2\cos x$



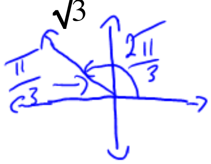
$y = 2\sec(x)$



period $= 2\pi$

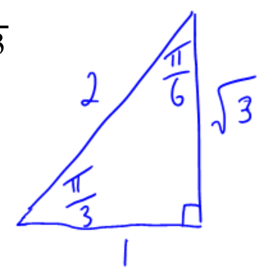
3. Determine the exact value of $\tan\frac{2\pi}{3}$

A. $-\sqrt{3}$ B. $-\frac{1}{\sqrt{3}}$ C. $\frac{1}{\sqrt{3}}$ D. $\sqrt{3}$



ref angle $= \frac{\pi}{3}$

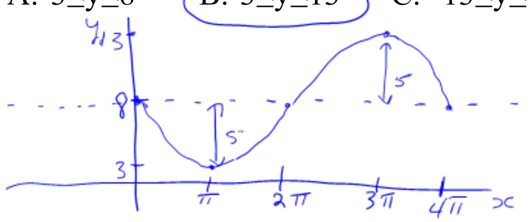
tan is neg in quad 2



$\therefore \tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$

4. Give the range of $y = -5\sin\frac{1}{2}x + 8$

A. 3≤y≤8 B. 3≤y≤13 C. -13≤y≤-3 D. -13≤y≤13



$p = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

range $3 \le y \le 13$

5. Simplify 4-8sin^2(6x)

A. cos 12x B. 2cos 6x C. 4cos 6x D. 4cos 12x

$\cos 2\theta = 1 - 2\sin^2\theta$

$4 - 8\sin^2(6x) = 4(1 - 2\sin^2(6x))$ $\theta = 6x$

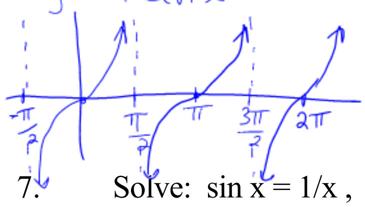
$= 4\cos(2(6x))$

$= 4\cos(12x)$

6. Determine an equation of an asymptote of y= -2 tan x

A. x=π/4 B. x= π/2 C. x= π D. x=2π

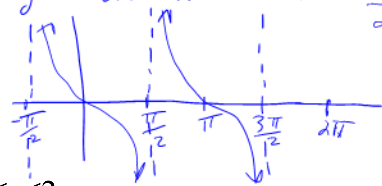
$y = \tan x$



$y = -2\tan x \Rightarrow -\frac{1}{2}y = \tan x$

vert exp 2

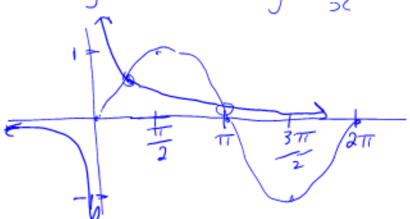
flip over x



7. Solve: sin x = 1/x , 0≤x≤2π

A. 0, 1.56 B. 1.11, 2.77 C. 3.44, 6.11 D. 0, 3.14, 6.28

$y = \sin x$ $y = \frac{1}{x}$



graphing calc

intersection points

at $x = 1.11, 2.77$

8.

At a seaport, the water has a maximum depth of 18 m at 3:00 am. After this maximum depth, the first minimum depth of 4 m occurs at 9:30 am. Assume that the relation between the depth, h metres, and the time, t hours, is a sinusoidal function. Determine an equation for h at any time t.

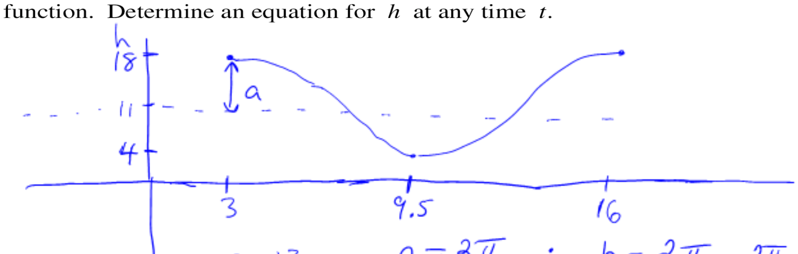
A. $h = 7\cos 2\pi\frac{(t-3)}{6.5} + 11$

B. $h = 7\cos 2\pi\frac{(t-3)}{13} + 11$

C. $h = 11\cos 2\pi\frac{(t-3)}{6.5} + 7$

D. $h = 11\cos 2\pi\frac{(t-3)}{13} + 7$

t	h
3	18
9.5	4



$a = 7$ $p = 13$ $p = \frac{2\pi}{b}$ $\therefore b = \frac{2\pi}{p} = \frac{2\pi}{13}$

$h_1 = 3$ $k = 11$

$h = a\cos(b(t - h_1)) + k$

$h = 7\cos\left(\frac{2\pi(t-3)}{13}\right) + 11$

9.

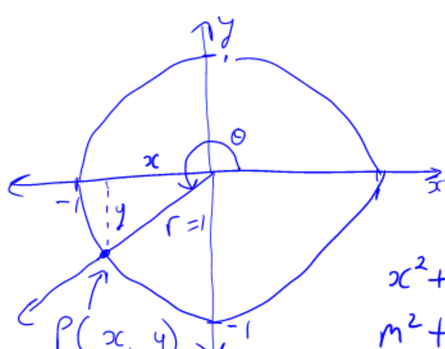
Point P is the intersection of the terminal arm of angle θ in standard position and the unit circle with centre $(0,\ 0)$. If P is in quadrant 3 and $\cos\theta = m$, determine the coordinates of P in terms of m.

A. $\left(-m,\ \sqrt{1-m^2}\right)$

B. $\left(-m,\ -\sqrt{1-m^2}\right)$

C. $\left(m,\ \sqrt{1-m^2}\right)$

D. $\left(m,\ -\sqrt{1-m^2}\right)$



$\cos\theta = m$

$\cos\theta = \frac{x}{r}$

$r = 1$

$\cos\theta = x$

$\therefore x = m$ $m < 0$

$x^2 + y^2 = r^2$

$m^2 + y^2 = 1^2$

$y^2 = 1 - m^2$

$y = \pm\sqrt{1-m^2}$

Quad 3

$\therefore y = -\sqrt{1-m^2}$

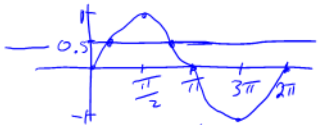
$\therefore P\left(m, -\sqrt{1-m^2}\right)$

10. Determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for:

$\sin(ax) = 0.5$, where a is an integer and $a \geq 1$

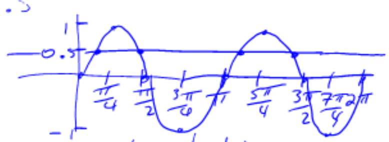
A. 2 B. a/2 C. a D. 2a

$a = 1$ $\sin x = 0.5$



2 solutions

$a = 2$ $\sin 2x = 0.5$



4 solutions

$\therefore$ number of solutions is $2a$

11. a) Solve algebraically, giving exact values for x, where $0 \leq x < 2\pi$.

$$2\cos^2 x - \cos x = 1$$

$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

b) Give the general solution for this equation. (Solve over the set of real numbers, giving exact value solutions.)

$2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

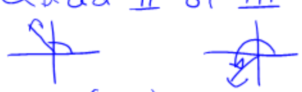
$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$\cos x = \frac{-1}{2}$ or $\cos x = 1$

Quad II or III

Quad I or IV



$R(\angle x) = \frac{\pi}{3}$

$R(\angle x) = 0$

$x = 0, 2\pi$

$\uparrow$ not included $0 \leq x < 2\pi$

$\therefore x = 0$

$x = \pi - \frac{\pi}{3}$ and $\pi + \frac{\pi}{3}$

$x = \frac{3\pi}{3} - \frac{\pi}{3}$ and $\frac{3\pi}{3} + \frac{\pi}{3}$

$x = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$

$$\boxed{\begin{array}{l} \text{general solution: } x = \frac{2\pi}{3} + 2\pi n, \ n \in \mathbb{Z} \\ x = \frac{4\pi}{3} + 2\pi n, \ n \in \mathbb{Z} \\ x = 0 + 2\pi n, \ n \in \mathbb{Z} \end{array}}$$

12. Prove the identity: $\dfrac{\cos\theta+\cot\theta}{1+\sin\theta}=\cot\theta$

$$\frac{\frac{\sin\theta}{1}\cdot\left(\frac{\cos\theta}{1}+\frac{\cos\theta}{\sin\theta}\right)}{\frac{\sin\theta}{1}\cdot\left(\frac{1}{1}+\frac{\sin\theta}{1}\right)}$$

$$\frac{\sin\theta\cos\theta+\cos\theta}{\sin\theta+\sin^2\theta}$$

$$\frac{\cos\theta\,(\cancel{\sin\theta+1})}{\sin\theta\,(\cancel{1+\sin\theta})}$$

$$\frac{\cos\theta}{\sin\theta}$$

$$\frac{\cos\theta}{\sin\theta}$$

13. Convert 120˚ to radians.

A. $2\pi/3$ B. $5\pi/6$ C. $3\pi/2$ D. $6\pi/5$

$120° \times \frac{\pi}{180°} = \frac{120\pi}{180} = \frac{12\pi}{18} = \frac{2\pi}{3}$

14. Determine the amplitude of $y = -2\sin 3(x - \frac{\pi}{4}) + 4$

$y = a\sin b(x-h)+k$
amplitude $= |a|$

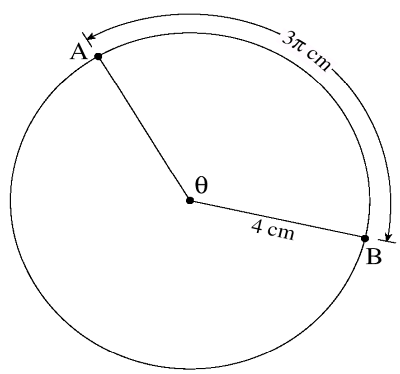
$|-2| = 2 = $ amp

A. -2 B. 2 C. 3 D. 4

15.

A circle has a radius of 4 cm. If the length of arc AB shown on the diagram is 3π cm, determine the measure of the central angle θ in radians.

$a = r\theta$



$\theta = \frac{a}{r}$ ← arc length, ← radius

$\theta = \frac{3\pi}{4}$

A. $\frac{3\pi}{4}$

B. $\frac{4}{3\pi}$

C. $\frac{3\pi}{2}$

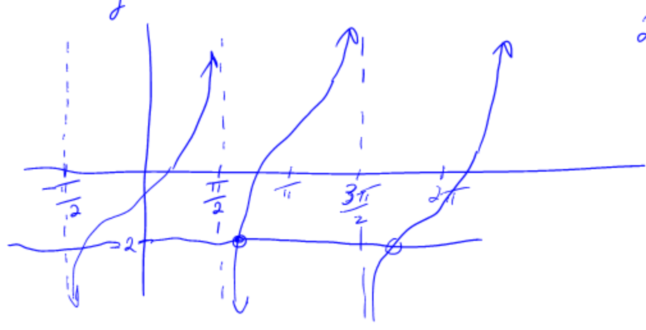
D. 3π

16. Solve: tan x - cos x = -2 , $0 \le x \le 2\pi$

A. 1.17, 4.10
B. 1.97, 5.32
C. 1.17, 1.57, 4.10, 4.71
D. 1.57, 1.97, 4.71, 5.32

$y = \tan x - \cos x$
$y = -2$

by graphing calc
2 intersections at
$x = 1.97, 5.32$



17.

Solve: $4\cos^2 x = 3$, $0 \le x < 2\pi$

$\cos^2 x = \frac{3}{4}$ $\quad$ $\cos x = \pm\sqrt{\frac{3}{4}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

A. $\frac{\pi}{6}, \frac{11\pi}{6}$

B. $\frac{\pi}{3}, \frac{5\pi}{3}$

C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\cos x = \frac{\sqrt{3}}{2}$

Quad I or IV

$R(\angle x) = \frac{\pi}{6}$

$x = \frac{\pi}{6}, \; 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \; \frac{12\pi}{6} - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \; \frac{11\pi}{6}$

$\cos x = -\frac{\sqrt{3}}{2}$

Quad II or III

$R(\angle x) = \frac{\pi}{6}$

$x = \pi - \frac{\pi}{6}, \; \pi + \frac{\pi}{6}$

$x = \frac{6\pi}{6} - \frac{\pi}{6}, \; \frac{6\pi}{6} + \frac{\pi}{6}$

$x = \frac{5\pi}{6}, \; \frac{7\pi}{6}$

18. Determine an expression equivalent to tan ß + cot ß.

A. 1 $\quad$ B. sin ß cos ß $\quad$ C. sec ß csc ß $\quad$ D. sin ß + cos ß

$$\tan B + \cot B = \frac{\sin B}{\cos B} + \frac{\cos B}{\sin B}$$

$$= \frac{\sin^2 B}{\sin B \cos B} + \frac{\cos^2 B}{\sin B \cos B}$$

$$= \frac{\sin^2 B + \cos^2 B}{\sin B \cos B}$$

$$= \frac{1}{\sin B \cos B}$$

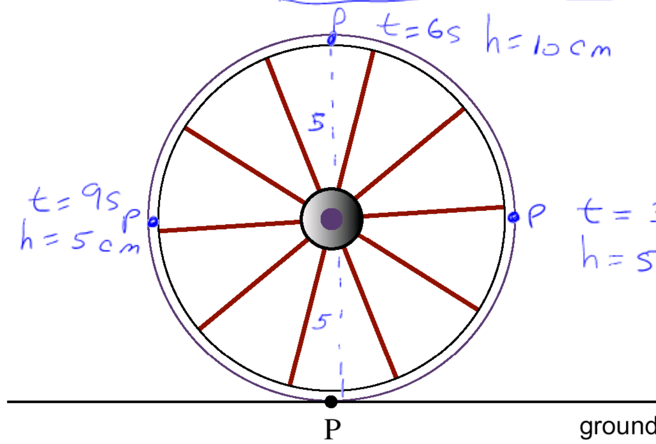
$$= \csc B \sec B$$

$$= \sec B \csc B$$

19.

A wheel with diameter 10 cm is rolling along the ground. Point P on the edge of the wheel is on the ground as shown in the diagram at time $t = 0$ seconds. Which equation gives the height, h, of point P above the ground at time t seconds, if the wheel rotates once every 12 seconds?

t	h
0	0
3	5
6	10
9	5
12	0



$t = 0$s $h = 0$cm

$t = 12$s $h = 0$cm

→ period = 12s

$P = \frac{2\pi}{b}$

$b = \frac{2\pi}{P}$

$b = \frac{2\pi}{12}$

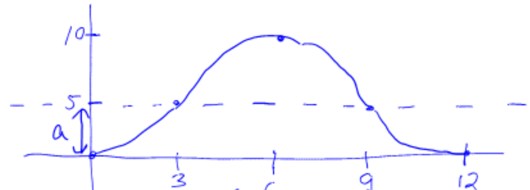
$b = \frac{\pi}{6}$

A. $h = -5\cos\frac{\pi}{12}t$

B. $h = -5\cos\frac{\pi}{6}t$

C. $h = -5\cos\frac{\pi}{12}t + 5$

D. $h = -5\cos\frac{\pi}{6}t + 5$



$h = a\cos(b(t - h_1)) + k$

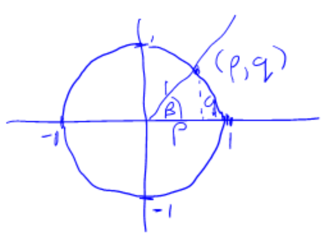
$h = -5\cos\frac{\pi t}{6} + 5$

$h_1 = 0$

$k = 5$

20. The point (p, q) is the point of intersection of the terminal arm of angle ß in standard position and the unit circle centered at (0,0). Which expression represents cscß?

A. p B. q C. 1/p D. 1/q



$\sin\beta = \frac{q}{r}$

$\therefore \csc\beta = \frac{r}{q} = \frac{1}{q}$

21. Which expression is equivalent to 6 sin(8x)cos(8x)?

A. sin(8x) B. sin(16x) C. 3sin(4x) D. 3sin(16x)

$\sin 2\theta = 2\sin\theta\cos\theta$

$6\sin(8x)\cos(8x) = 3(2\sin(8x)\cos(8x))$ $\theta = 8x$

$= 3(\sin(2(8x)))$

$= 3\sin(16x)$

22.

Determine the equations of the asymptotes of the function $y = \tan bx$, where $b > 0$.

A. $x = \frac{n\pi}{b}$, n is an integer

B. $x = \frac{n\pi}{2b}$, n is an integer

C. $x = \frac{\pi}{b} + \frac{n\pi}{b}$, n is an integer

D. $x = \frac{\pi}{2b} + \frac{n\pi}{b}$, n is an integer

$y = \tan x$ has asymptotes at $x = \frac{\pi}{2} + \pi n$

$y = \tan(bx)$ has a horiz comp by $\frac{1}{b}$

$x \to bx$

$\therefore bx = \frac{\pi}{2} + \pi n$

and asymptotes are $\boxed{x = \frac{\pi}{2b} + \frac{\pi n}{b}}$

23. a) Solve algebraically, giving **exact values** for x, where $0 \le x \le 2\pi$.

$$2\sin^2 x = \sin x$$

$\boxed{x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi}$

b) Give the general solution for this equation.

$2\sin^2 x - \sin x = 0$

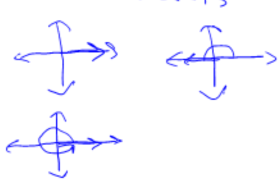
$\sin x (2\sin x - 1) = 0$

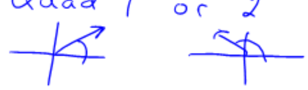
$\sin x = 0$ or $2\sin x - 1 = 0$

on x axis

$\sin x = \frac{1}{2}$

Quad 1 or 2





$R(\angle x) = \frac{\pi}{6}$

$x = 0, \pi, 2\pi$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{6\pi}{6} - \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

general Solution

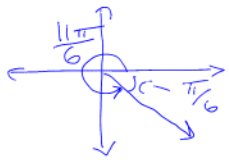
$x = \pi n, n \in \mathbb{Z}$

$x = \frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$

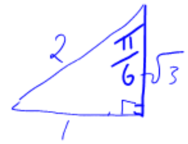
$x = \frac{5\pi}{6} + 2\pi n, n \in \mathbb{Z}$

24. Give the exact value of $\cos\frac{11\pi}{6}$

A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{2}}{2}$ D. $\frac{\sqrt{3}}{2}$



cos is pos in quad 4



$\therefore \cos\frac{11\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

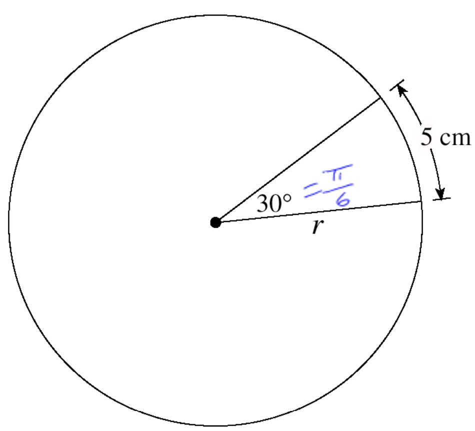
25. Simplify $\frac{2\sin\theta}{\sin 2\theta}$

A. 1 B. $\cos\theta$ C. $\csc\theta$ D. $\sec\theta$

$\frac{2\sin\theta}{\sin 2\theta} = \frac{\cancel{2\sin\theta}}{\cancel{2\sin\theta}\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$

26.

An arc of length 5 cm subtends an angle of $30°$ at the centre of a circle with radius r, as shown in the diagram. Determine the value of r.



$\theta = \frac{a}{r}$

$r = \frac{a}{\theta}$

$r = \frac{5}{\frac{\pi}{6}}$

$r = 9.55$

A. 4.77
B. 6.00
C. 9.55
D. 10.00

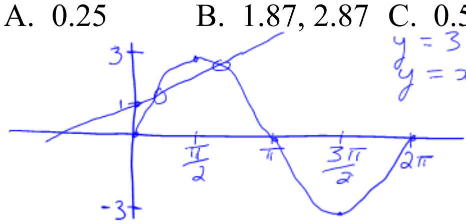
27. Determine the period of $y = \tan(\pi x)$

A. 1   B. 2   C. $\pi/2$ D. π

period $= \frac{\pi}{b} = \frac{\pi}{\pi} = 1$

28. Solve: $3\sin x = x + 1, \quad 0 \le x \le 2\pi$

A. 0.25   B. 1.87, 2.87   C. 0.54, 1.54   D. 0.54, 1.87



$y = 3\sin x$
$y = x + 1$

By graphing calc
$x = 0.54, 1.87$

29. Simplify: $\sin\left(\frac{3\pi}{2} + x\right)$

A. sin x   B. cos x   C. -sin x   D. -cos x

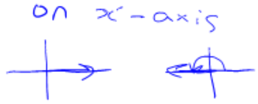
$\sin\left(\frac{3\pi}{2} + x\right) = \sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$
$= (-1)\cos x + (0)\sin x$
$= -\cos x$

30. Solve: $\sin^2 x = \sin x \cos x$, $0 \le x < 2\pi$

A. $x = 0, \frac{\pi}{4}$ B. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

C. $x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ D. $x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

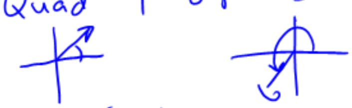
$\sin^2 x = \sin x \cos x$
$\sin^2 x - \sin x \cos x = 0$
$\sin x(\sin x - \cos x) = 0$
$\sin x = 0$ or $\sin x - \cos x = 0$
on x-axis $\sin x = \cos x$
graphing calc



$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

$\boxed{x = 0, \pi}$

or $\Rightarrow$ $\sin x = \cos x$
$\Rightarrow \frac{\sin x}{\cos x} = 1$
$\Rightarrow \tan x = 1$
Quad 1 or 3

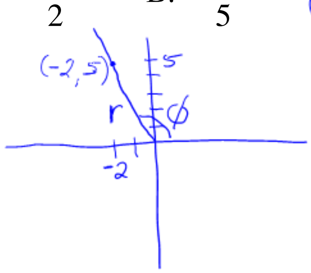


$R(\angle x) = \frac{\pi}{4}$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

31. The terminal arm of angle ¢ in standard position passes through the point (-2, 5). Determine the value of sec ¢.

A. $-\frac{\sqrt{21}}{2}$ B. $\frac{\sqrt{21}}{5}$ C. $-\frac{\sqrt{29}}{2}$ D. $\frac{\sqrt{29}}{5}$

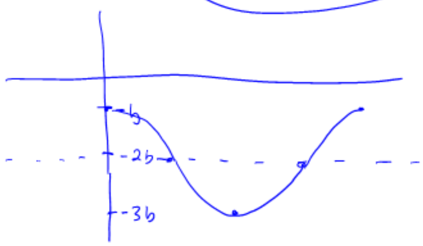


$r^2 = x^2 + y^2$
$r^2 = (-2)^2 + (5)^2$
$r^2 = 4 + 25$
$r^2 = 29$
$r = \sqrt{29}$

$\cos\phi = \frac{x}{r}$
$\sec\phi = \frac{r}{x}$
$\sec\phi = \frac{\sqrt{29}}{-2}$

32. Determine the range of the function: y=$b$cos(ax) -2b , where a>0, b>0.

A. b≤y≤3b B. -3b≤y≤-b C. b-a ≤y≤ b+a D. 2b - a ≤y≤ 2b + a



33.

Determine the general solution for: $\sin 2x = -\frac{1}{2}$

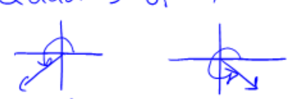
A. $\frac{7\pi}{12} + 2n\pi, \ \frac{11\pi}{12} + 2n\pi$ (n is any integer)

B. $\frac{7\pi}{12} + n\pi, \ \frac{11\pi}{12} + n\pi$ (n is any integer)

C. $\frac{13\pi}{12} + 2n\pi, \ \frac{21\pi}{12} + 2n\pi$ (n is any integer)

D. $\frac{13\pi}{12} + n\pi, \ \frac{21\pi}{12} + n\pi$ (n is any integer)

$\sin 2x = -\frac{1}{2}$
Quad 3 or 4



$R(\angle 2x) = \frac{\pi}{6}$
$p = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

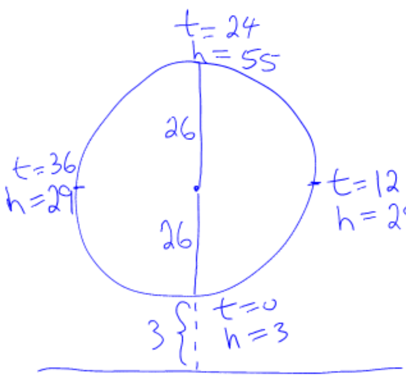
$\therefore 2x = \pi + \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$
$2x = \frac{6\pi}{6} + \frac{\pi}{6}, \quad \frac{12\pi}{6} - \frac{\pi}{6}$
$2x = \frac{7\pi}{6}, \quad \frac{11\pi}{6}$
$x = \frac{7\pi}{12}, \quad \frac{11\pi}{12}$

$\therefore$ general solution
$x = \frac{7\pi}{12} + \pi n$
$x = \frac{11\pi}{12} + \pi n$ n is any integer

34. A Ferris wheel has a radius of 26 m and its centre is 29 m above the ground. It rotates once every 48 seconds. Sandy gets on the Ferris wheel at its lowest point, and then the wheel starts to rotate.

a) Determine a sinusoidal equation that gives Sandy's height, *h*, above the ground as a function of the elapsed time, *t*, where *h* is in metres and *t* is in seconds.

b) Determine the first time *t* (in seconds), when Sandy will be 36 m above the ground.



$p = 48s$

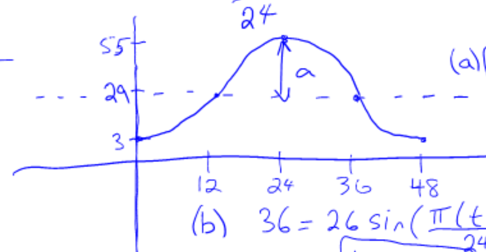
$p = \frac{2\pi}{b}$

$b = \frac{2\pi}{p}$

$b = \frac{2\pi}{48s}$

$b = \frac{\pi}{24}$

t(s)	h(m)
0	3
12	29
24	55
36	29
48	3



$h = a\sin(b(t-h_1)) + k$

(a) $\boxed{h = 26\sin\left(\frac{\pi(t-12)}{24}\right) + 29 \quad \text{or} \quad h = 26\cos\left(\frac{\pi(t-24)}{24}\right) + 29}$

(b) $36 = 26\sin\left(\frac{\pi(t-12)}{24}\right) + 29$

$\Rightarrow \boxed{t = 14.1s}$

35. Find an equivalent form for the following: $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x}$

Common denominator $\Rightarrow (1-\sin x)(1+\sin x)$

A. $\tan^2 x$ B. $\cot^2 x$ C. $2\tan^2 x$. D. $2\cot^2 x$

$\dfrac{\sin x(1+\sin x)}{(1-\sin x)(1+\sin x)} - \dfrac{\sin x(1-\sin x)}{(1-\sin x)(1+\sin x)} =$

$\dfrac{\sin x + \sin^2 x}{1-\sin^2 x} - \dfrac{\sin x - \sin^2 x}{1-\sin^2 x} =$

$\dfrac{\sin x + \sin^2 x - \sin x + \sin^2 x}{1-\sin^2 x} =$

$\dfrac{2\sin^2 x}{\cos^2 x} =$

$2\tan^2 x$

36. Convert $\frac{5\pi}{3}$ radians to degrees. Do NOT use a calculator.

A. 60° B. 120° C. 300° D. 330°

$\frac{5\pi}{3} \times \frac{180^\circ}{\pi} = 300^\circ$

37. Solve: sin 2x - cos x =1 , $0 \le x \le 2\pi$

A. 0, 5.07 B. 3.14, 4.32 C. 3.14, 4.36 D. 0.42, 1.89, 2.95, 4.21

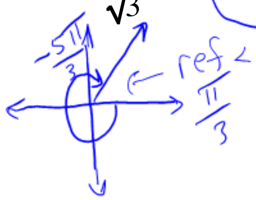
graph $y = \sin 2x - \cos x$
and $y = 1$
and find intersection points using graphing calc

intersection points at $x = 3.14, 4.36$

38. Determine the exact value of $\cot \frac{-5\pi}{3}$. Do NOT use a calculator.

$\cot \frac{-5\pi}{3} = \frac{1}{\tan \frac{-5\pi}{3}}$

A. $-\frac{1}{\sqrt{3}}$ B. $\frac{1}{\sqrt{3}}$ C. $-\sqrt{3}$ D. $\sqrt{3}$



tan is positive in Quad I

$\therefore \tan \frac{-5\pi}{3} = \tan \frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\therefore \cot \frac{-5\pi}{3} = \frac{1}{\tan \frac{-5\pi}{3}} = \frac{1}{\sqrt{3}}$

39. Determine the period of the function $f(x) = -\frac{1}{2}\sin\frac{x}{3}$.

A. $\frac{2\pi}{3}$ B. π C. 4π D. 6π

$f(x) = \frac{-1}{2} \sin \frac{x}{3}$

$= \frac{-1}{2} \sin \frac{1}{3} x$

In $f(x) = a \sin(b(x-h)) + k$

period $= \frac{2\pi}{b}$

$\therefore$ period $= \frac{2\pi}{\frac{1}{3}} = 6\pi$

40. Solve: 2 sin x + 1 = 0 , $0 \le x \le 2\pi$

A. $-\frac{\pi}{6}, -\frac{5\pi}{6}$ B. $\frac{\pi}{6}, \frac{5\pi}{6}$ C. $\frac{7\pi}{6}, \frac{11\pi}{6}$ D. $\frac{4\pi}{3}, \frac{5\pi}{3}$

$\sin x = \frac{-1}{2}$

Quad III or IV

$R(\angle x) = \frac{\pi}{6}$

$\therefore x = \pi + \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6}$

$x = \frac{6\pi}{6} + \frac{\pi}{6}, \ \frac{12\pi}{6} - \frac{\pi}{6}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

41. Determine an equivalent form for the following: $\dfrac{\tan\theta \csc^2\theta}{\sec^2\theta}$

A. $\tan\theta$ B. $\cot\theta$ C. $\tan^2\theta$ D. $\tan^3\theta$

$\dfrac{\tan\theta \csc^2\theta}{\sec^2\theta} =$

$\dfrac{\frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\sin^{\cancel{2}}\theta}}{\frac{1}{\cos^2\theta}} =$

$\dfrac{\frac{1}{\cos\theta \sin\theta}}{\frac{1}{\cos^2\theta}} = \frac{1}{\cancel{\cos}\theta \sin\theta} \cdot \frac{\cos^{\cancel{2}}\theta}{1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

42. Simplify: cos(π - 2x)

A. -cos 2x B. -sin 2x C. cos 2x D. sin 2x

$\cos(\pi - 2x) = \cos\pi\cos 2x + \sin\pi \sin 2x$
$= (-1)\cos 2x + (0)\sin 2x$
$= -\cos 2x$

43.

A wheel with radius 20 cm has its centre 30 cm above the ground. It rotates once every 15 seconds. Determine an equation for the height, h, above the ground of a point on the wheel at time t seconds if this point has a maximum height at $t = 2$ seconds.

$p = 15\text{ s}$ ∴ the point moves $\frac{1}{4}$ of the circle in 3.75 s

$p = \frac{2\pi}{b}$

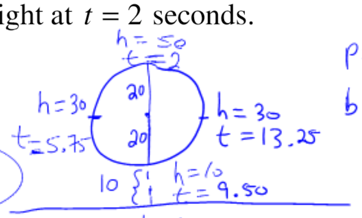
$b = \frac{2\pi}{p} = \frac{2\pi}{15}$

A. $h = 20\cos\frac{2\pi}{15}(t+2)+30$

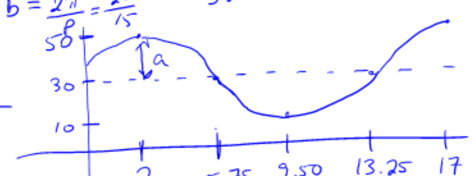
B. $h = 20\cos\frac{2\pi}{15}(t-2)+30$

C. $h = 30\cos\frac{2\pi}{15}(t+2)+20$

D. $h = 30\cos\frac{2\pi}{15}(t-2)+20$



t(s)	h(m)
2	50
5.75	30
9.50	10
13.25	30
17	50



$h = a\cos(b(t - h_1)) + k$

$\boxed{h = 20\cos\left(\frac{2\pi(t-2)}{15}\right) + 30}$

44.

Determine a cosine equation that has the following general solution: $\frac{\pi}{2}+n\pi$, $\frac{\pi}{6}+2n\pi$, $\frac{11\pi}{6}+2n\pi$, where n is an integer.

A. $\cos x\left(2\cos x+\sqrt{2}\right)=0$

B. $\cos x\left(2\cos x+\sqrt{3}\right)=0$

C. $\cos x\left(2\cos x-\sqrt{2}\right)=0$

D. $\cos x\left(2\cos x-\sqrt{3}\right)=0$

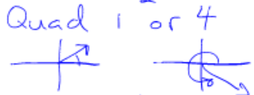
$\cos x\left(2\cos x - \sqrt{3}\right) = 0$

$\cos x = 0$ or $2\cos x - \sqrt{3} = 0$

$\cos = \frac{\sqrt{3}}{2}$

on y-axis

Quad 1 or 4



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

general ⇒ $\boxed{x = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}}$

$R(\angle x) = \frac{\pi}{6}$

$x = \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6},\ \frac{12\pi}{6} - \frac{\pi}{6}$

$x = \frac{\pi}{6},\ \frac{11\pi}{6}$

gen ⇒ $\boxed{x = \frac{\pi}{6} + 2\pi n,\ x = \frac{11\pi}{6} + 2\pi n \quad n \in \mathbb{Z}}$

Try $\frac{\pi}{2}, \frac{\pi}{6}, \frac{11\pi}{6}$ in the 4 equations above to lead you to the right answer!

45. Solve the following equation algebraically.

$3\cos^2 x + \cos x - 2 = 0, \qquad 0 \le x \le 2\pi$

$(3\cos x - 2)(\cos x + 1) = 0$

$3\cos x - 2 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{2}{3}$ $\qquad \cos x = -1$

Quad 1 or 4 $\qquad$ On neg x axis

$R(\angle x) = 0.84$ $\qquad x = \pi$

$x = 0.84,\ 2\pi - 0.84$

$x = 0.84,\ 5.44$

46. Prove the identity: $(\csc\theta - \sin\theta)\tan\theta = \dfrac{\sin 2\theta}{2\sin\theta}$

$\csc\theta\tan\theta - \sin\theta\tan\theta$

$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} - \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$

$\frac{1 - \sin^2\theta}{\cos\theta}$

$\frac{\cos^2\theta}{\cos\theta}$

$\cos\theta$

$\frac{\cancel{2\sin\theta}\cos\theta}{\cancel{2\sin\theta}}$

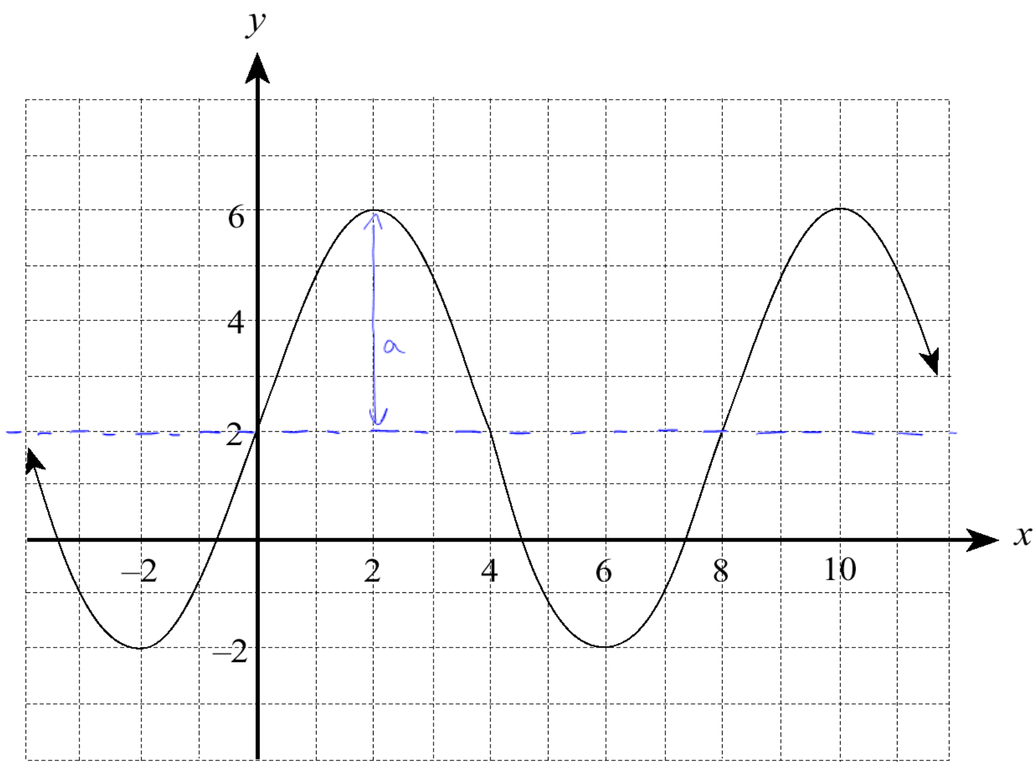
$\cos\theta$

47. Evaluate to 2 decimal places: sec 0.89 ← in radians because no "degrees" sign

$\frac{1}{\cos 0.89}$

Ans: 1.59

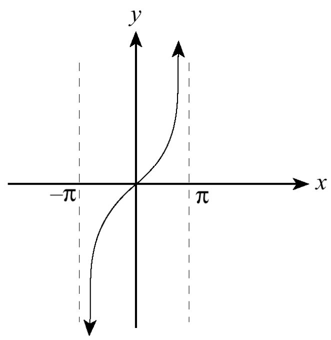
48. Determine the amplitude of the following graph:



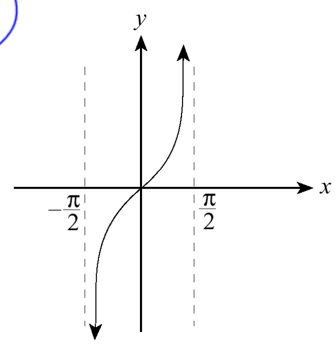
A. 2 B. 4 C. 6 D. 8

49. Which graph shows one period of y= tan x ?

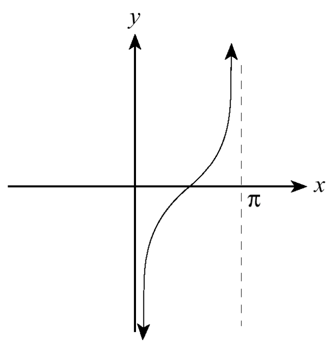
A.



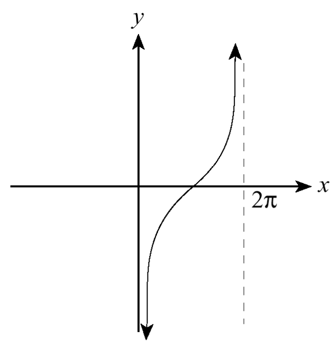
B.



C.



D.



50. Convert *m* radians to degrees.

A. $\frac{\pi}{180m}^\circ$ B. $\frac{\pi m}{180}^\circ$ C. $\frac{180}{\pi m}^\circ$ D. $\frac{180m}{\pi}^\circ$

$m \times \frac{180^\circ}{\pi} = \frac{180m^\circ}{\pi}$

51. Simplify $\cos(\frac{3\pi}{2} + \beta)$ $= \cos\frac{3\pi}{2}\cos\beta - \sin\frac{3\pi}{2}\sin\beta$

$= (0)\cos\beta - (-1)\sin\beta = \sin\beta$

A. sin ß B. -sin ß C. cos ß D. -cos ß

52. Determine the phase shift of the function $f(x) = -3\sin(2x - \frac{\pi}{6}) + \frac{\pi}{2}$

A. π/12 B. π/6 C. π/3 D. π/2

$f(x) = -3\sin\left(2\left(x - \frac{\pi}{12}\right)\right) + \frac{\pi}{2}$

phase shift $\frac{\pi}{12}$

53. Which expression is equivalent to $(\sin^2 ß - \cos^2 ß)^2 - \sin^2 2ß$?

A. $-2\sin^2 2ß$ B. $2\sin^2 2ß$ C. $-\cos 4ß$ D. $\cos 4ß$

$$(\sin^2\beta - \cos^2\beta)^2 - \sin^2 2\beta =$$
$$(\cos 2\beta)^2 - \sin^2 2\beta =$$
$$\cos^2 2\beta - \sin^2 2\beta =$$
$$\cos(2(2\beta)) =$$
$$\cos(4\beta)$$

54. Solve for x: $2\tan^2 x - 5\tan x = 3$, where $0 \le x \le 2\pi$ (accurate to 2 decimal places)

$$2\tan^2 x - 5\tan x - 3 = 0$$
$$(2\tan x + 1)(\tan x - 3) = 0$$
$$2\tan x + 1 = 0 \quad \text{or} \quad \tan x - 3 = 0$$
$$\tan x = \frac{-1}{2} \quad \text{or} \quad \tan x = 3$$

Quad 2 or 4

$R(\angle x) = 0.46$

$x = \pi - 0.46,\ 2\pi - 0.46$

$x = 2.68,\ 5.82$

Quad I or 3

$R(\angle x) = 1.25$

$\therefore x = 1.25,\ \pi + 1.25$

$x = 1.25,\ 4.39$

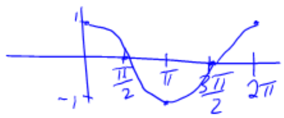
55.

For what value of x is the following expression undefined?

$$\frac{\sin x}{1+\cos x}, \text{ where } 0 \le x < 2\pi$$

where $1+\cos x = 0$

$\cos x = -1$



A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

56. Simplify: $\dfrac{\sqrt{\sec^2 - 1}}{\sqrt{\csc^2 - 1}} = \dfrac{\sqrt{\tan^2 x}}{\sqrt{\cot^2 x}} = \dfrac{\tan x}{\cot x} = \dfrac{\tan x}{\frac{1}{\tan x}}$

A. $\tan^2 x$ B. $\cot^2 x$ C. $\tan^4 x$ D. $\cot^4 x$

$= \tan x \cdot \dfrac{\tan x}{1}$

$= \tan^2 x$

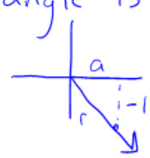
57.

Determine the value of $\sec\theta$ if $\cot\theta = -a$, where $a > 0$ and $\sin\theta < 0$.

cot is neg means tan is neg ↓

tan is neg is Quad 2 or 4 Quad 3 or 4

angle is in quad 4



$\cot\theta = \dfrac{x}{y} = \dfrac{a}{-1}$

$r^2 = a^2 + (-1)^2$

$r^2 = a^2 + 1$

$r = \sqrt{a^2+1}$

$\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{a^2+1}}{a}$

A. $\dfrac{\sqrt{a^2+1}}{a}$

B. $-\dfrac{\sqrt{a^2+1}}{a}$

C. $\dfrac{a+1}{a}$

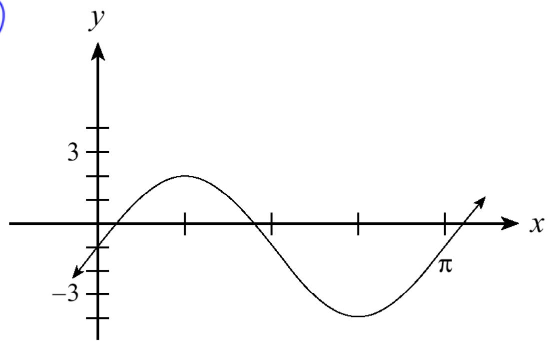
D. $-\dfrac{a+1}{a}$

58. Which of the following represents the graph of y= 3sin 2x - 1 ? amp = 3

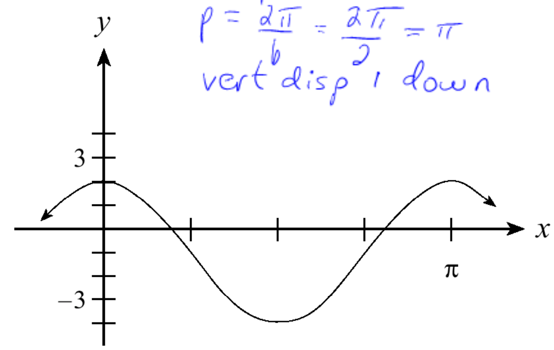
$p = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

vert disp 1 down

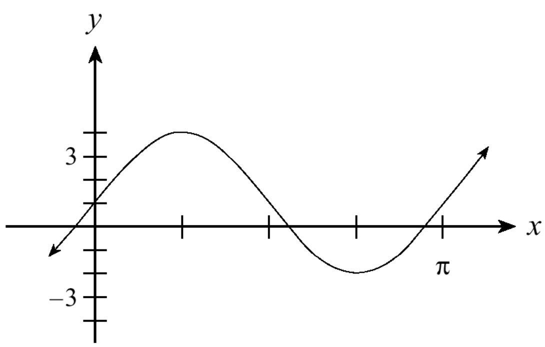
A.



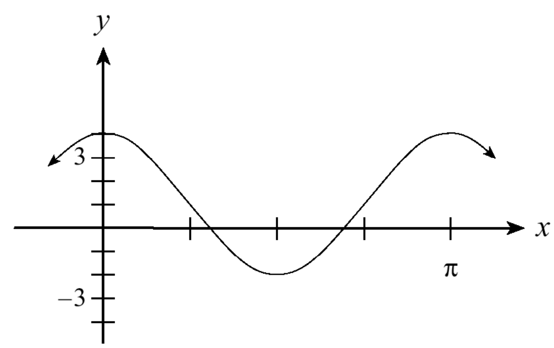
B.



C.



D.



59. How many solutions does $\sin^2 x = 1/3$ have over the interval $0 \le x \le 2\pi$?

A. 1 B. 2 C. 3 D. 4

$\sin^2 x = \frac{1}{3}$

$\sin x = \frac{\pm 1}{\sqrt{3}}$

$\sin x = \frac{1}{\sqrt{3}}$

Quad 1 or 2

$R(\angle x) = 0.62$

$x = 0.62, 2.53$

$\sin x = -\frac{1}{\sqrt{3}}$

Quad 3 or 4

$R(\angle x) = 0.62$

$x = 3.76, 5.67$

4 solutions

60.

Which expression is equivalent to $\dfrac{2\tan\theta\cot 2\theta}{1+\tan\theta}$?

A. $1-\tan\theta$

B. $\dfrac{1}{1-\tan\theta}$

C. $\dfrac{1}{1+\tan\theta}$

D. $\dfrac{1}{\tan\theta}$

$$\frac{2\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos 2\theta}{\sin 2\theta}\right)}{1+\tan\theta}$$

$$\frac{\frac{2\sin\theta(\cos^2\theta-\sin^2\theta)}{2\sin\theta\cos^2\theta}}{1+\tan\theta}$$

$$\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{1+\tan\theta}$$

$$\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta(1+\tan\theta)}$$

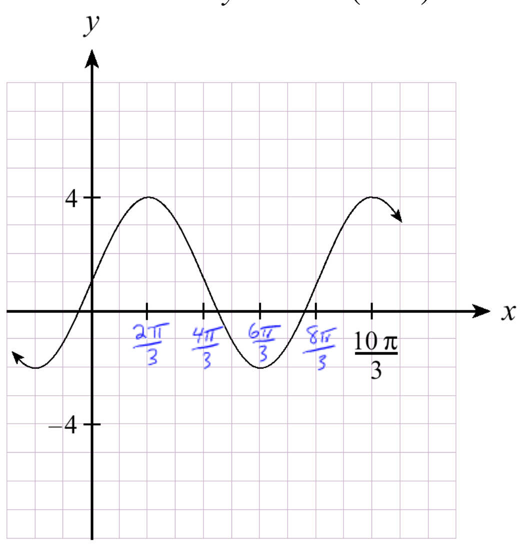
$$\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\cos\theta\sin\theta}$$

$$\frac{(\cancel{\cos\theta+\sin\theta})(\cos\theta-\sin\theta)}{\cos\theta(\cancel{\cos\theta+\sin\theta})}$$

$$\frac{\cos\theta-\sin\theta}{\cos\theta}$$

$$1-\tan\theta$$

61. The function $y = a\cos b(x-c)+d$ is graphed below. Determine b.



$p=\frac{2\pi}{b}$

$b=\frac{2\pi}{p}=\frac{2\pi}{\frac{8\pi}{3}}=\frac{6\pi}{8\pi}=\frac{6}{8}=\frac{3}{4}$

A. 3/5

B. 3/4

C. $\dfrac{8\pi}{3}$

D. $\dfrac{10\pi}{3}$

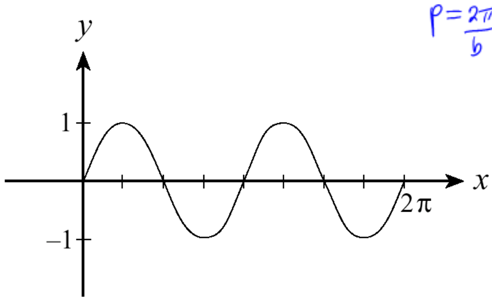
62.

Which of the following is the graph of $y = -\sin 2x$, for $0 \le x \le 2\pi$?

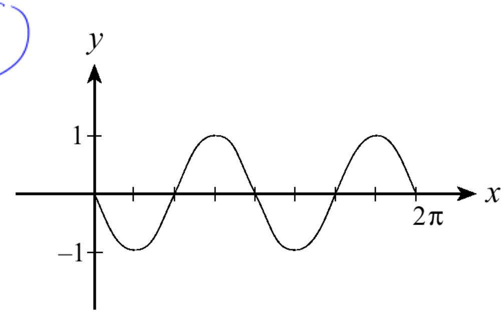
amp $= 1$

$p = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

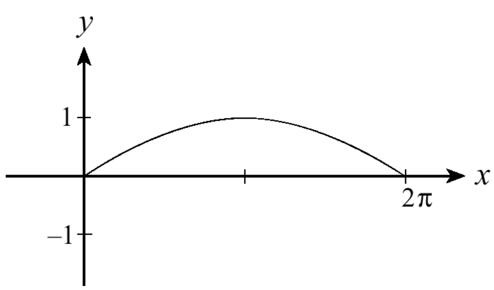
A.



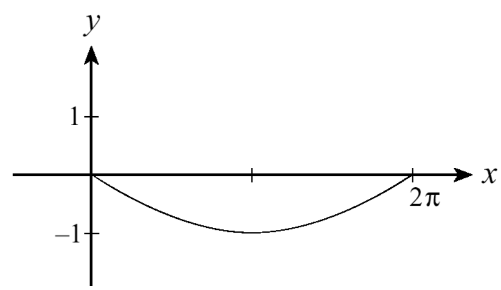
B.



C.

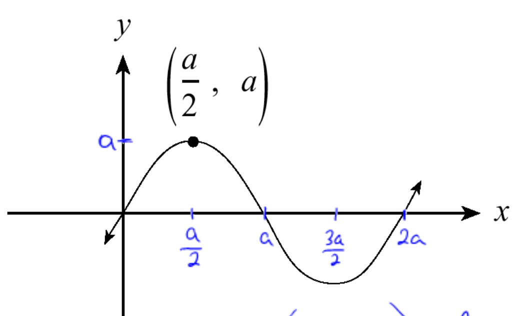


D.



63.

Determine the equation of the following sine curve.



$y = a\sin(b(x-c)) + d$

$c = 0,\ d = 0$

$a = a,\quad b = \frac{2\pi}{P} = \frac{2\pi}{2a} = \frac{\pi}{a}$

$y = a\sin\left(\frac{\pi}{a}x\right)$

A. $y = a\sin\frac{\pi}{a}x$

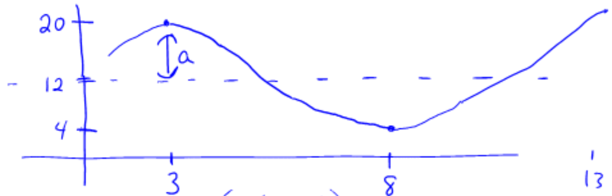
B. $y = a\sin\frac{a}{\pi}x$

C. $y = \frac{a}{2}\sin\frac{\pi}{a}x$

D. $y = \frac{a}{2}\sin\frac{2a}{\pi}x$

64.

A cosine curve has a maximum point at $(3,\ 20)$ and the nearest minimum point to the right of this point is $(8,\ 4)$. Which of the following is an equation for this curve?



A. $y = 8\cos\dfrac{2\pi}{5}(x+3)+12$

B. $y = 8\cos\dfrac{2\pi}{5}(x-3)+12$

C. $y = 8\cos\dfrac{\pi}{5}(x+3)+12$

D. $y = 8\cos\dfrac{\pi}{5}(x-3)+12$

$y = a\cos(b(x-c))+d$

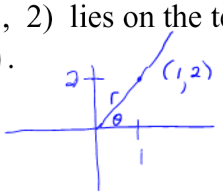
$a=8,\quad b=\dfrac{2\pi}{p}=\dfrac{2\pi}{10}=\dfrac{\pi}{5}$

$c=3,\quad d=12$

$y=8\cos\left(\dfrac{\pi}{5}(x-3)\right)+12$

65.

If the point $(1,\ 2)$ lies on the terminal arm of angle θ in standard position, determine the value of $\cos(\pi+\theta)$.



$r^2=x^2+y^2$

$r^2=1^2+2^2$

$r^2=5$

$r=\sqrt{5}$

A. $\dfrac{-2}{\sqrt{5}}$

B. $\dfrac{-1}{\sqrt{5}}$

C. $\dfrac{1}{\sqrt{5}}$

D. $\dfrac{2}{\sqrt{5}}$

$\cos(\pi+\theta) = \cos\pi\cos\theta - \sin\pi\sin\theta$

$= (-1)\cos\theta - (0)\sin\theta$

$= -\cos\theta$

$\cos\theta = \dfrac{x}{r} = \dfrac{1}{\sqrt{5}}$

$\therefore\ -\cos\theta = \dfrac{-1}{\sqrt{5}}$

Scholarship Questions! Nasty, with big fangs and sharp teeth! Be careful!

66. Prove the identity: $\dfrac{\sin 2x}{1-\cos 2x} = 2\csc 2x - \tan x$

$$\frac{2\sin x\cos x}{1-(1-2\sin^2 x)}$$

$$\frac{2\sin x\cos x}{2\sin^2 x} =$$

$$\frac{\cos x}{\sin x}$$

$$\cot x$$

$$\frac{2}{\sin 2x} - \frac{\sin x}{\cos x}$$

$$\frac{2}{2\sin x\cos x} - \frac{\sin x}{\cos x}$$

$$\frac{1}{\sin x\cos x} - \frac{\sin^2 x}{\sin x\cos x}$$

$$\frac{1-\sin^2 x}{\sin x\cos x}$$

$$\frac{\cos^2 x}{\sin x\cos x}$$

$$\frac{\cos x}{\sin x}$$

$$\cot x$$

67. Solve algebraically for x: $4\sin^2 x = 3\tan^2 x - 1$, $0 \le x \le 2\pi$. Give your answers as exact solutions.

$$\cos^2 x \cdot \left(4\sin^2 x\right) = \left(\frac{3\sin^2 x}{\cos^2 x} - 1\right) \cdot \cos^2 x$$

$$4\sin^2 x\cos^2 x = 3\sin^2 x - \cos^2 x$$

$$4\sin^2 x\left(1-\sin^2 x\right) = 3\sin^2 x - \left(1-\sin^2 x\right)$$

$$4\sin^2 x - 4\sin^4 x = 3\sin^2 x - 1 + \sin^2 x$$

$$\cancel{4\sin^2 x} - 4\sin^4 x = \cancel{4\sin^2 x} - 1$$

$$-4\sin^4 x = -1$$

$$4\sin^4 x - 1 = 0$$

$$\left(2\sin^2 x + 1\right)\left(2\sin^2 x - 1\right) = 0$$

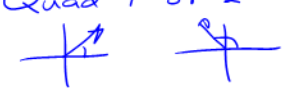
$2\sin^2 x + 1 = 0$ or $2\sin^2 x - 1 = 0$

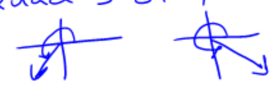
$\sin^2 x = \frac{-1}{2}$ $\qquad$ $\sin^2 x = \frac{1}{2}$

no solution $\qquad$ $\sin x = \frac{\pm 1}{\sqrt{2}}$

$\sin x = \frac{1}{\sqrt{2}}$ $\qquad$ $\sin x = \frac{-1}{\sqrt{2}}$

Quad 1 or 2 $\qquad$ Quad 3 or 4





$R(\angle x) = \frac{\pi}{4}$ $\qquad$ $R(\angle x) = \frac{\pi}{4}$

$x = \frac{\pi}{4}, \ \pi - \frac{\pi}{4}$ $\qquad$ $x = \pi + \frac{\pi}{4}, \ 2\pi - \frac{\pi}{4}$

$x = \frac{\pi}{4}, \ \frac{4\pi}{4} - \frac{\pi}{4}$ $\qquad$ $x = \frac{4\pi}{4} + \frac{\pi}{4}, \ \frac{8\pi}{4} - \frac{\pi}{4}$

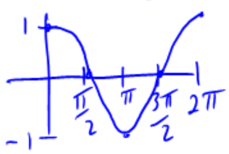
$\boxed{x = \frac{\pi}{4}, \ \frac{3\pi}{4}}$ $\qquad$ $\boxed{x = \frac{5\pi}{4}, \ \frac{7\pi}{4}}$

68. Solve algebraically: $\sin 2x = 2\cos x \cos 2x$, $0 \le x \le 2\pi$. Give your answer to two decimal places.

$$2\sin x\cos x = 2\cos x(1 - 2\sin^2 x)$$
$$2\sin x \cos x = 2\cos x - 4\sin^2 x\cos x$$
$$4\sin^2 x\cos x + 2\sin x\cos x = 2\cos x$$
$$4\sin^2 x\cos x + 2\sin x\cos x - 2\cos x = 0$$
$$2\cos x(2\sin^2 x + \sin x - 1) = 0$$
$$2\cos x(2\sin x - 1)(\sin x + 1) = 0$$

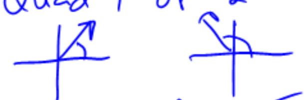
$2\cos x = 0$ or $2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$ or $\sin x = -1$



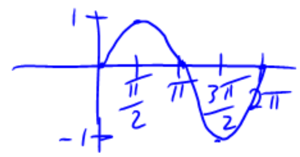
$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Quad 1 or 2



$R(\angle x) = \pi/6$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$



$x = \frac{3\pi}{2}$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$\boxed{x = 0.52, 1.57, 2.62, 4.71}$$

69. Change the equation $y = 6\sin x\cos^3 x + 6\sin^3 x\cos x - 3$ to the form y=AsinBx + D , where A, B, and D are constants. $y = 6\sin x\cos x\left(\cos^2 x + \sin^2 x\right) - 3$

$y = 6\sin x\cos x - 3$

$y = 3\left(2\sin x\cos x\right) - 3$

$y = 3\sin 2x - 3$

70. Solve algebraically: $\sec x + \tan^2 x - 3\cos x = 2$, $0 \le x \le 2\pi$. (Accurate to 2 decimal places).

$$\sec x \cdot \left(\sec x + (\sec^2 x - 1) - \frac{3}{\sec x} \right) = (2) \cdot \sec x$$

$$\sec^2 x + \sec^3 x - \sec x - 3 = 2\sec x$$

$$\sec^3 x + \sec^2 x - 3\sec x - 3 = 0$$

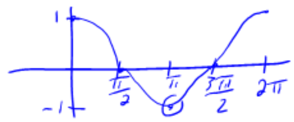
$$\sec^2 x (\sec x + 1) - 3(\sec x + 1) = 0$$

$$(\sec x + 1)(\sec^2 x - 3) = 0$$

$\sec x + 1 = 0$ or $\sec^2 x - 3 = 0$

$\sec x = -1$ $\qquad$ $\sec^2 x = 3$

$\cos x = -1$ $\qquad$ $\frac{1}{\cos^2 x} = 3$



$\cos^2 x = \frac{1}{3}$

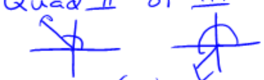
$\cos x = \frac{\pm 1}{\sqrt{3}}$

$x = \pi$

$x = 3.14$

$\cos x = \frac{1}{\sqrt{3}}$ $\qquad$ $\cos x = \frac{-1}{\sqrt{3}}$

Quad I or 4 $\qquad$ Quad II or III



$R(\angle x) = 0.96$ $\qquad$ $R(\angle x) = 0.96$

$x = 0.96, 2\pi - 0.96$ $\qquad$ $x = \pi - 0.96, \pi + 0.96$

$x = 0.96, 5.32$ $\qquad$ $x = 2.18, 4.10$

71. a) Prove the identity: $\dfrac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = \dfrac{1}{1+\sec x}$

b) Solve the following equation for x, $0 \le x \le 2\pi$.

$$\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = -\frac{1}{3}$$, accurate to at least 2 decimal places.

(a)

$$\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} \qquad = \qquad \frac{1}{1+\sec x}$$

$$\frac{\cos^2 x - 2\cos x}{(\cos x + 1)(\cos x - 2)} \qquad \qquad \frac{1}{\left(1+\frac{1}{\cos x}\right)} \cdot \frac{\cos x}{\cos x}$$

$$\frac{\cos x(\cancel{\cos x - 2})}{(\cos x + 1)(\cancel{\cos x - 2})} \qquad \qquad \frac{\cos x}{\cos x + 1}$$

$$\frac{\cos x}{\cos x + 1}$$

(b)

$$\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = \frac{-1}{3}$$

$$\frac{\cos^2 x - 2\cos x}{(\cos x + 1)(\cos x - 2)} = \frac{-1}{3}$$

$$\frac{\cos x(\cancel{\cos x - 2})}{(\cos x + 1)(\cancel{\cos x - 2})} = \frac{-1}{3}$$

$$\frac{\cos x}{\cos x + 1} = \frac{-1}{3}$$

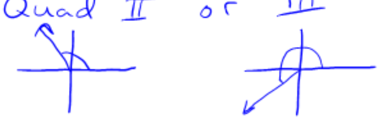
$$3\cos x = -1(\cos x + 1)$$

$$3\cos x = -\cos x - 1$$

$$4\cos x = -1$$

$$\cos x = \frac{-1}{4}$$

Quad II or III



$R(\angle x) = 1.32$

$x = \pi - 1.32,\ \pi + 1.32$

$x = 1.82,\ 4.46$

72. If the infinite geometric series $1 - \sin x + \sin^2 x - \sin^3 x + \ldots$ has a sum of 2, what is the smallest positive value of x? Give your answer in radians.

$a = 1$

$r = -\sin x$

$$S = \frac{a}{1-r}$$

$$\frac{1}{1+\sin x} = 2$$

$$1 = 2 + 2\sin x$$

$$2\sin x = -1$$

$$\sin x = \frac{-1}{2}$$

Quad 3 or 4

$$R(\angle x) = \frac{\pi}{6}$$

$$x = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$$

$$x = \frac{6\pi}{6} + \frac{\pi}{6},\ \frac{12\pi}{6} - \frac{\pi}{6}$$

$$x = \frac{7\pi}{6},\ \frac{11\pi}{6}$$

smallest positive x is $\frac{7\pi}{6}$

73. Prove the following identity: $\frac{1+\cos 2\theta}{2} = \cos^2\theta$

$$\frac{1+2\cos^2\theta - 1}{2}$$

$$\frac{2\cos^2\theta}{2}$$

$$\cos^2\theta$$

74. Prove the following identity: $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

$\cos 4\theta$

$\cos(2\theta + 2\theta)$

$\cos 2\theta \cos 2\theta - \sin 2\theta \sin 2\theta$

$(2\cos^2\theta - 1)(2\cos^2\theta - 1) - (2\sin\theta\cos\theta)^2$

$4\cos^4\theta - 2\cos^2\theta - 2\cos^2\theta + 1 - (2\sin\theta\cos\theta)^2$

$4\cos^4\theta - 4\cos^2\theta + 1 - 4\sin^2\theta\cos^2\theta$

$4\cos^4\theta - 4\cos^2\theta + 1 - 4(1 - \cos^2\theta)\cos^2\theta$

$4\cos^4\theta - 4\cos^2\theta + 1 - 4\cos^2\theta + 4\cos^4\theta$

$8\cos^4\theta - 8\cos^2\theta + 1$

75. Solve algebraically. Give your answer to 2 decimal places.

$3 \sin x = 4 + 4 \csc x$ $\qquad\qquad 0 \le x \le 2\pi$

$\sin x \cdot (3 \sin x) = \left(4 + \frac{4}{\sin x}\right) \cdot \sin x$

$3\sin^2 x = 4 \sin x + 4$

$3\sin^2 x - 4 \sin x - 4 = 0$

$(3 \sin x + 2)(\sin x - 2) = 0$

$3 \sin x + 2 = 0$ or $\sin x - 2 = 0$

$\sin x = -\frac{2}{3}$ $\qquad$ $\sin x = 2$

no solution

Quad 3 or 4

$R(\angle x) = 0.73$

$x = \pi + 0.73,\ 2\pi - 0.73$

$x = 3.87,\ 5.55$

76. a) For what values of x in the interval $0 \le x \le 2\pi$ is the following equation **undefined**?

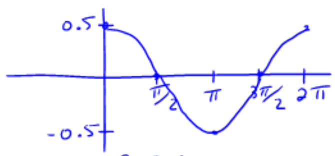
$$\log_3\left(\frac{\cos x}{2}\right) - \log_3(\cos^2 x) = 1$$

b) Solve the above equation for x ($0 \le x \le 2\pi$). (Accurate to 2 decimal places)

(a) x values such that $\frac{\cos x}{2} \le 0$ and

x values such that $\cos^2 x \le 0$

$y = \frac{\cos x}{2} = \frac{1}{2}\cos x$

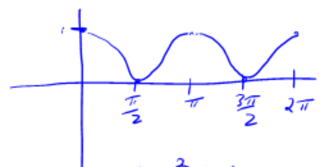


$\frac{\cos x}{2} \le 0$

for

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$y = \cos^2 x$



$\cos^2 x \le 0$

for

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore$ equation is undefined for $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

(b) $\log_3\left(\frac{\cos x}{2}\right) - \log_3\left(\cos^2 x\right) = 1$

$\log_3\left(\frac{\frac{\cos x}{2}}{\cos^2 x}\right) = 1$

$\log_3\left(\frac{\cos x}{2\cos^2 x}\right) = 1$

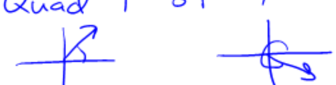
$\frac{\cos x}{2\cos^2 x} = 3$

$\frac{1}{2\cos x} = 3$

$1 = 6\cos x$

$\cos x = \frac{1}{6}$

Quad 1 or 4



$R(\angle x) = 1.40$

$x = 1.40,\ 2\pi - 1.40$

$x = 1.40,\ 4.88$

Answers:

1. c
2. d
3. a
4. b
5. d
6. b
7. b
8. b
9. d
10. d
11. a) $x = \frac{2\pi}{3} or \frac{4\pi}{3} or 0$

    b) $x = \frac{2\pi}{3} + 2n\pi$

    $x = \frac{4\pi}{3} + 2n\pi$

    $x = 0 + 2n\pi$
12. see solution sheet
13. a
14. b
15. a
16. b
17. c
18. c
19. d
20. d
21. d
22. d
23. a) $x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

    b) $x = n\pi, n \in I$

    $x = \frac{\pi}{6} + 2n\pi, n \in I$

    $x = \frac{5\pi}{6} + 2n\pi, n \in I$
24. d
25. d
26. c
27. a
28. d
29. d
30. d
31. c
32. b
33. b
34. a)

    $h(t) = -26\cos(\frac{\pi}{24}t) + 29$

    b) 14.1 s
35. c
36. c
37. c
38. b
39. d
40. c
41. b
42. a
43. b
44. d
45. x=0.84, 3.14, 5.44
46. see solution sheet
47. 1.59
48. b
49. b
50. d
51. a
52. a
53. d
54. x=1.25, 2.68, 4.39, 5.82
55. c
56. a
57. a
58. a
59. d
60. a
61. b
62. b
63. a
64. d
65. b
66. see solution sheet
67. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
68. x=0.52, 1.57, 2.62, 4.71
69. $y = 3\sin(2x) - 3$
70. x=.96, 2.19,3.14, 4.10, 5.33
71. a) see solution sheet

    b)x=1.82, 4.46
72. $x = \frac{7\pi}{6}$
73. see solution sheet
74. see solution sheet
75. x=3.87, 5.55
76. a)undefined for $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ b)1.40, 4.88

Note to teachers:

The questions here come from a variety of sources. Some come from Alberta provincial exams, or are based on questions from those documents. The scholarship questions at the end come from the BC scholarship exams from 1991 to 1996. Most of the multiple choice questions are based on provincial exams from 1994-1996, but I have tried to change the numbers where formatting was not too large an issue. I haven't put many trig identities on the written sections. If you want more trig identities, go to the BCAMT website and download Mark Garneau's outstanding list of all of the trig identities from provincial exams (it goes back a dozen years or so).

I generally hand this out at the beginning of the unit (including the answer key), and I collect it the day of the test. I flip through the booklet just to see if there is writing on each page, and I give the students a few marks. During the unit, I have a few photocopied solution manuals (showing all my steps) floating around the class as well. Students can sign them out and take them home if they wish.

If you find any errors in the answer key, or have any suggestions that I could add, feel free to email me at kdueck@sd42.ca and I'll be happy to reply.

Kelvin Dueck
Pitt Meadows Secondary

PS Thanks to Gretchen McConnell for helping with error checking my answer keys! It's been much appreciated!